Lap Time Simulation

James Hakewill, January 2000

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Introduction

The goal of this exercise has been to create a program or system that would represent a reasonably accurate model of a racing car travelling around an arbitrary circuit. This would allow experimentation to take place with a variety of factors that are likely to affect the performance.

The work carried out thus far has concentrated on building a simple model that can produce a lap time based upon a number factors, including:

a) a set of XY co-ordinates for the circuit
b) the power output of the car's engine
c) the weight of the car
d) the coefficient of friction of the tyres
e) the frontal area and Cd of the vehicle

A number of assumptions have been made thus far that are likely to affect the outcome, to a greater or lesser degree:

a) The coefficient of friction of the tyre remains constant at all times

   - in practice, this tails off as the load placed on the tyre increases, and hence brings into play the design of the suspension in terms of the effects of weight transfer, and the possible benefits of adding downforce.

b) The sample points on the circuit are close enough together to calculate the car's performance statically at each point. Acceleration between sample points is taken to be constant.

c) The driver is capable of getting the maximum performance from the tyres during braking and cornering (or is able to achieve a fixed percentage of the maximum possible forces).

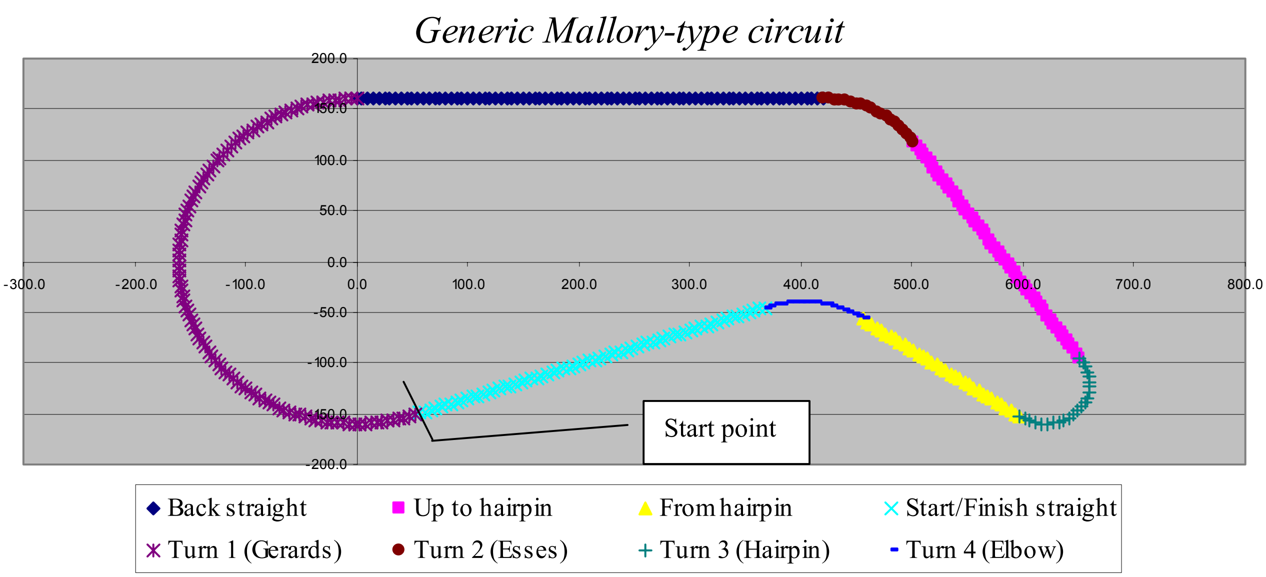
d) The circuit is completely flat – no hills or dips are present.

   – in practice the presence of hills will affect the ability of the car to brake and accelerate, as well as the grip available from the tyres for cornering.

The Circuit

In order to produce a lap time, first we need a circuit. As a starting point, a model has been created for a generic circuit, somewhat similar to Mallory Park in Leicestershire. It is anticipated that at some future point, real circuit data will be available either from on-car telemetry systems, or possibly from computer game simulations of circuits, such as the popular F1GP, for which track editors are available.

The circuit is assumed to be completely flat, in order to make for simpler calculations. It is possible that Z data could be incorporated at a later date, however most current racing car data logging systems which can provide track maps only work in two dimensions (X and Y).



From the starting point at the entry to the main corner (Gerards), the 2.04km circuit is arranged as follows:

Sector	*Length (m)*	*Radius (m)*	*Arc*
Turn 1 (Gerards)	556	160	199°
Back straight (Revett)	420	-	-
Turn 2 (Esses)	96	100	55°
Straight to hairpin	260	-	-
Turn 3 (Shaws Hairpin)	114	40	164°
Straight from hairpin	171	-	-
Turn 4 (Devils Elbow)	93	100	53°
Start/Finish straight	332	-	-

Compared with the real Mallory Park, the Esses here are a single corner rather than an S-shaped one, and the hairpin is probably too large a radius. In addition, all the corners have a constant radius. A racing driver would make all efforts to increase the radius of a corner at entry and exit, to increase speed when possible.

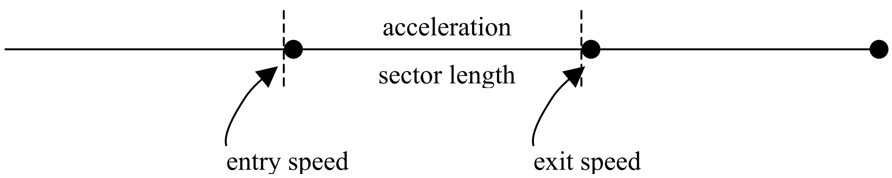
It is important to remember when modelling new circuits that the XY co-ordinates to be used should describe the path taken by the car, the *racing line*, rather than the centre line of the paved tarmac area.

The XY co-ordinates are given in metres, relative to the origin in the centre of turn 1, as can be seen from the track map above. The co-ordinates occur at intervals of approximately 5m along the circuit, however consistent spacing between points is not critical since the length of each sector is calculated individually.

Description of sectors

The circuit is split up into many individual sectors – around 400 in the case of our generic circuit, each sector is around 5m in length.

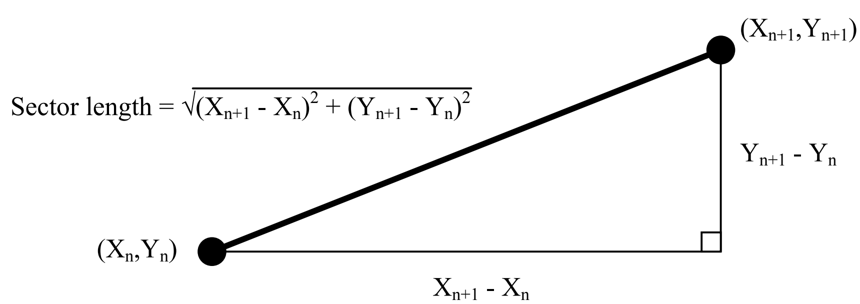
For each sector, we want to be able to calculate the exit speed at the end of the sector, and the time taken to traverse the sector. Clearly we want to maximise speed at all points, within the limits of the car, in order to minimise the time taken.



If the car is accelerating from rest, we can take account of the available grip from the tyres, the engine power and aerodynamic drag and calculate the acceleration possible at the start of each new sector. From this, we can then calculate the exit speed at the end of the sector, given the length of the sector and the assumption that the acceleration is constant.

We take a sector to be the length of track following a particular point.

If we take two points (X_n,Y_n) and (X_{n+1},Y_{n+1}), then the distance between them – the length of sector n – can be determined as follows:



The angle A from the horizontal can be found by:

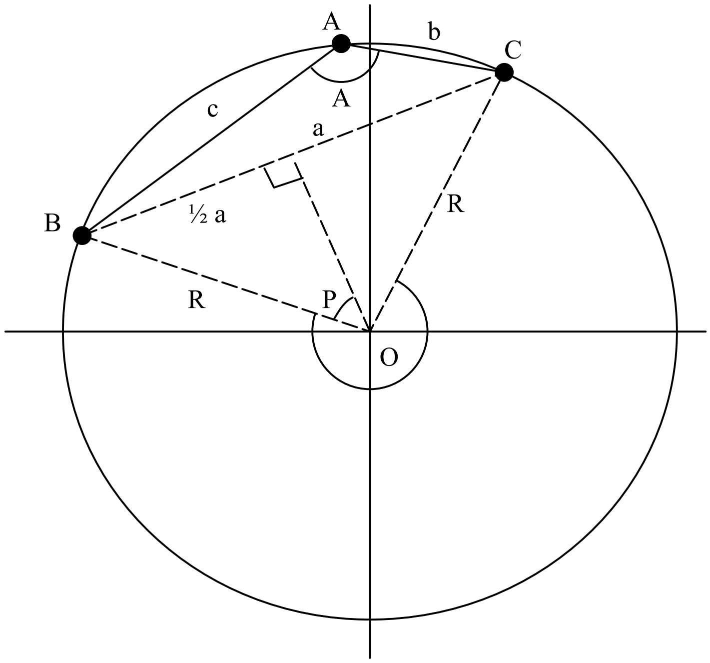
$$\tan A = (Y_{n+1} - Y_n) / (X_{n+1} - X_n)$$

We need to model some complex behaviour, where the car accelerates from rest, brakes to the entry point of a corner, negotiates the corner at the maximum possible speed, and accelerates again at the exit of the corner. We will get on to the details of the calculations shortly, but first we need to look at how we can determine corner radius from the sequence of XY co-ordinates, since this will determine our maximum speed around the corner for a given car.

Calculations

Calculating corner radius

Given three XY co-ordinates, it is possible to determine the radius of the circle that passes through all of the points – the effective corner radius at that point. The points do not even need to be evenly spaced on the curve.



Having drawn three points A, B, C on a circle whose centre is at O, we know from a circle theorem that the exterior angle BOC is twice the interior angle A (BAC).

"The angle which an arc of a circle subtends at the centre of a circle is twice that subtended by the arc at any other point on the circumference of the circle."

In triangle BOC, we know that the two radial sides are of equal length, and thus we can split this isosceles triangle into two right angle triangles. Angle P will be half the interior angle BOC:

$$P = (360 - 2A) / 2 = 180 - A$$

We then determine the radius R using trigonometry:

$$\sin P = ½ a / R$$

$$\begin{aligned} R &= a / (2 \sin P) \\ &= a / (2 \sin (180 - A)) \end{aligned}$$

The angle A can be found using the cosine rule:

$$\cos A = (b^2 + c^2 - a^2) / 2bc$$

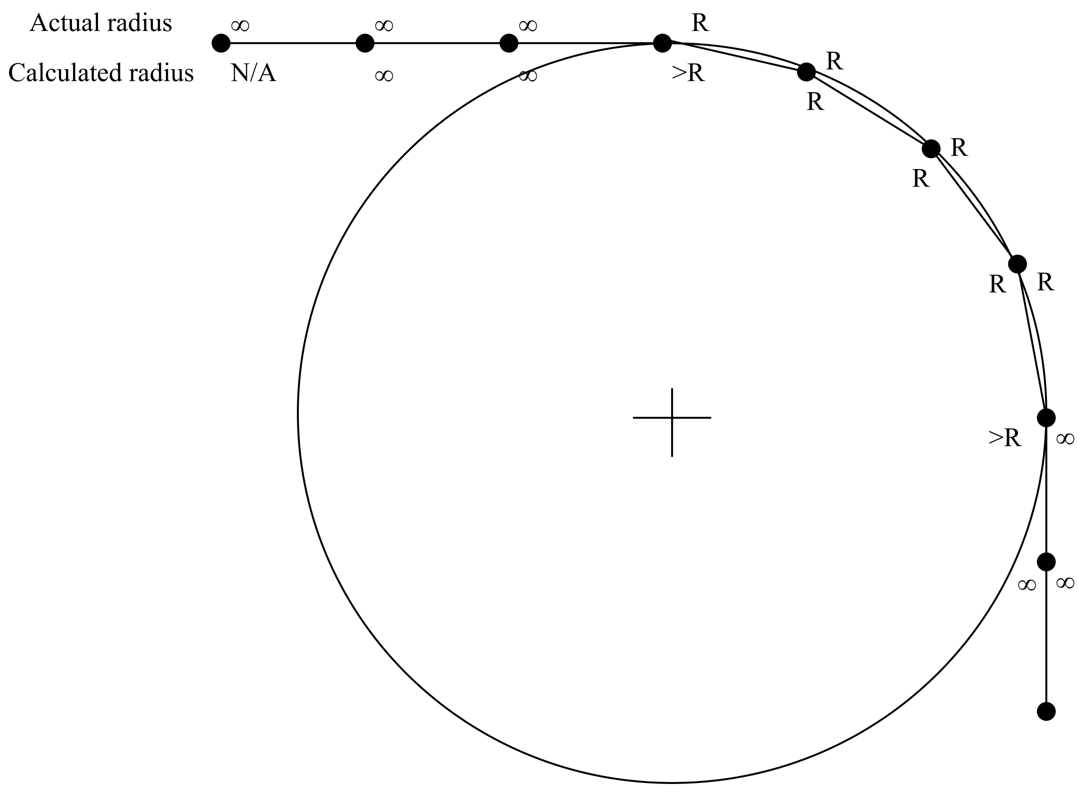
The distances a, b and c can be found easily from the XY co-ordinates for points A, B and C.

We will use this method to calculate the effective radius at point A (start point of current sector), taking point B from the start point of the previous sector, and point C from the start point of the next sector.

Corner entry and exit transition

The diagram below shows a set of co-ordinates that describe a path around a 90° bend of radius R, with a straight before and after.

Since we have created the example, it is possible to write onto the diagram the actual corner radius for each sector, beside each start point. We will be calculating the effective corner radius for a particular sector by using three points – the start point of the sector, the end point of the sector (next sector start point), and the start point of the previous sector. This has some effects at the start and end of corners that need to be understood.



a) The first sector of the corner will be of a larger radius than the actual radius of the corner.

b) The first sector of the following straight will not be infinity, as would be expected, but have be a radius larger than the actual corner radius.

We could change the three points which are used to calculated the effective corner radius for a sector, but we will always get these effects on the transition between corner and straight, but in different places.

Maximum corner speed

It is useful for us to be able to calculate the fastest speed the car could travel though a particular sector of a known radius. We can then determine the braking effort required for the corner, and detect when the driver should balance the car on the throttle at the maximum corner speed.

We will assume that each sector is of a constant radius, and at the maximum 'limiting' speed, the speed will remain constant through the corner.

The maximum speed will be determined by the ability of the tyres to turn the car through the corner and simultaneously push it forward through the air against drag to maintain a constant speed.

The maximum force that can be transmitted through the tyre without slippage is given by:

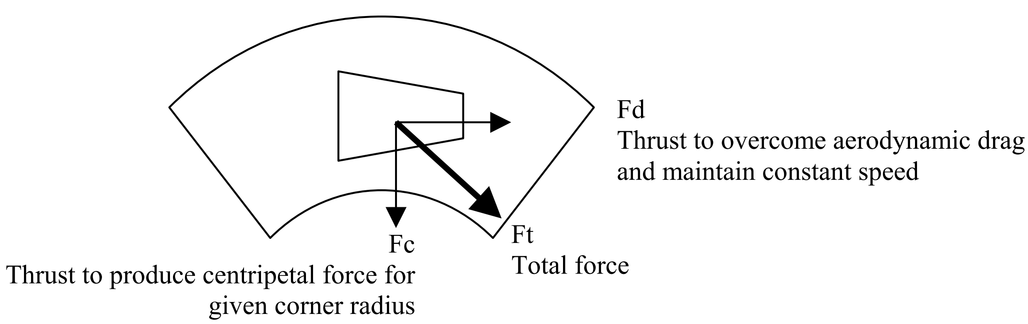
$$F = \mu R$$

Where μ is the coefficient of friction of the tyre, and R is the 'normal force' acting on the tyre. Note that he area of the tyre's contact patch is not part of the equation. We can therefore keep life simple by modelling the car as having only one tyre.

At rest, the normal force will be equal to the weight of the car (mass x gravity). At speed the normal force will be affected by the aerodynamics of the car – if it produces downforce, this will lead to the normal force being larger, and if it generates lift at speed (a bad thing), then the normal force will be reduced.

Carroll Smith describes in his book *Tune to Win* how the coefficient of friction changes with tyre slip angle, and tails off as the normal force is increased. For the time being, we will assume that the coefficient of friction remains constant, and will ignore the effects of suspension geometry and weight transfer between the corners that will affect the normal force on each tyre.

The forces that must be transmitted through the tyre as it travels around the corner are:



When the car is cornering at the maximum speed, the total force Ft will be equal to the maximum force that can be transmitted through the tyres:

$$Ft = \sqrt{Fc^2 + Fd^2} = \mu R$$

We will use a coefficient of friction for a racing tyre of $\mu = 1.4$, quoted from *Tune to Win* and *Competition Car Downforce.*

Aerodynamic drag

The aerodynamic drag force is determined by:

a) The coefficient of drag of the car (Cd)
b) The frontal area of the car A (Note Cd for whole car is relative to frontal area)
c) The density of air, ρ (1.19 kg/m^3)
d) The square of the velocity, v^2

$$Fd = Cd \times \tfrac{1}{2}\rho v^2 \times A$$

Allan Staniforth's book *Race and Rally Car Source Book* contains a table of Cd figures for a variety of cars, and includes figures for an unnamed Formula Ford 1600. Adjusted for m^2:

Cd	*Frontal area m^2*	*Note*
0.525	1.6 x 0.91 = 1.46	Max X x Max Y dimension
0.71	1.11	Outline area

The two numbers appear to reflect the different drag coefficients that need to be used depending on how the frontal area is measured, since very similar results are produced by multiplying these values for Cd and area.

Using the formula above will give us a drag figure for a car without wings. Clearly if we choose to add wings, the drag produced by these will need to be added to the total drag force, based on the wing's Cd and plan area. (Values of Cd for wings are normally quoted relative to the plan area).

Centripetal force

This is the force required to keep the car turning rather than travelling straight ahead. It is given by:

$$Fc = (mv^2)/r$$

Where *m* is the mass, *v* is the velocity and *r* the radius of the circle.

Combined force

We can now put the two together and say:

$$\begin{aligned} Ft^2 &= (Fc^2 + Fd^2) \\ &= (m^2v^4/r^2) + ({}^1/_4\rho^2v^4A^2Cd^2) \\ &= v^4((m^2/r^2) + ({}^1/_4\rho^2A^2Cd^2)) \end{aligned}$$

Equation for maximum velocity

If we combine with the limiting friction equation, we can solve for the velocity v:

$$(\mu R)^2 = v^4((m^2/r^2) + ({}^1/_4\rho^2A^2Cd^2))$$

$$v^4 = (\mu R)^2 / ((m^2/r^2) + ({}^1/_4\rho^2A^2Cd^2))$$

$$v = \sqrt[4]{(\mu R)^2 / ((m/r)^2 + ({}^1/_2\rho ACd)^2)}$$

This equation tells us the maximum speed the car can go round a given radius curve. The maths would however be further complicated if the car had some downforce, since the normal force R would depend on the square of velocity v.

Acceleration from rest

The power/torque of the engine, drive train efficiency, the mass of the vehicle, the tyre coefficient of friction and the aerodynamic drag will determine the car's acceleration from rest and ultimate top speed in a straight line.

The accelerative force produced by the engine can be determined from the engine power output and the speed. The useful power at the wheels will be the engine power at the crankshaft reduced by the drive train efficiency.

Force = Power x Velocity

The bhp figure can easily be converted to a useful figure in Watts.

Useful power P in Watts = 7457 x Brake Horsepower x Drive Train efficiency

A typical figure for drive train efficiency in a Formula Ford car might be 91% (*Competition Car Downforce*).

Aerodynamic drag will be as before, $Fd = Cd \times \frac{1}{2}\rho v^2 \times A$

The total accelerative force at velocity v will therefore be:

$$Fa = (P/v) - \tfrac{1}{2}\rho v^2 ACd$$

Since the force will be zero when the car is at rest (effectively the engine is not turning), we need to fudge the operation of the clutch by starting the car at some speed slightly greater than zero.

We could alternatively calculate the force by using a torque figure for the engine, along with the drive train gearing ratio, and the effective radius of the driving wheels. Effective radius will be the rolling circumference divided by 2Π.

Acceleration can be calculated using the equation below, where *m* is the mass of the object, and F the force acting upon it.

$$a = F/m$$

Given current velocity *u* and acceleration a, the equation below will provide the velocity *v* after distance *s* has been travelled:

$$v^2 = u^2 + 2as$$

We can combine these equations to allow us to calculate the speed of the car at the end of a sector of length *s*, given an entry speed *u* for the sector:

$$a = ((P/u) - \tfrac{1}{2}\rho u^2 Acd)/m$$

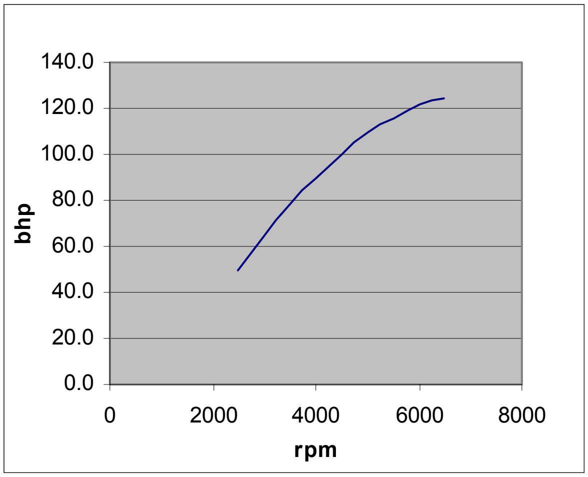
$$v^2 = u^2 + 2s((P/u) - \tfrac{1}{2}\rho u^2 Acd)/m$$

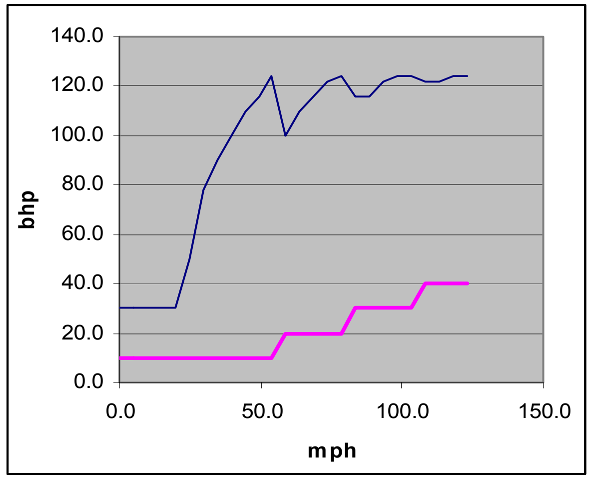
$$v = \sqrt{u^2 + 2s((P/u) - \tfrac{1}{2}\rho u^2 Acd)/m}$$

Note that we have used the entry speed *u* to calculate the maximum accelerative force available on entry to the sector (it was referred to as *v* in the previous equations).

Engine Power

If we have a plot of engine power against revs, and we know the effective radius of the driving wheels, final drive and gear ratios of the car in question we can select the gear that the car should be in at any particular speed. This would enable us to determine a plot of the power output for that speed.





Again, this method could also be used given the torque curve of the engine. It is difficult to model the effect of gear change time using this method. When the engine is not accelerating the car, the car will slow down the revs will drop, and oscillation can occur on upshifts unless some hysteresis is introduced.

We can use this technique at any point to determine the available power from the engine based on u, the entry speed to the sector.

An alternative technique might be to model the driver's behaviour. The driver starts off from rest in 1st gear, then when the 'shift-up' light comes on, selects the next gear. When the revs drop past a certain point, due to braking, then a lower gear is selected. In addition to the data required above, this method requires an 'upshift point' and a 'downshift point' to be selected. The power available can then be determined from the revs of the engine at any point.

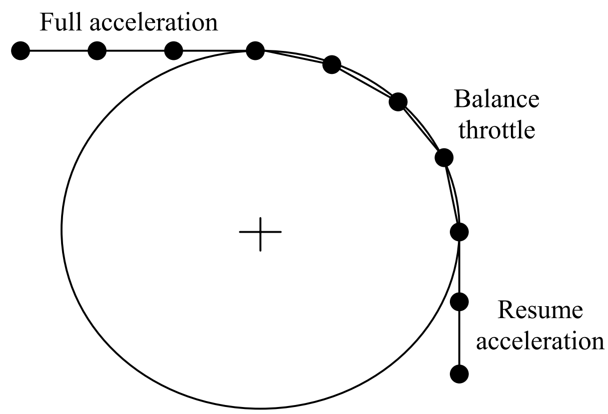
We currently model the time taken for the driver to change gear using the second method. When a gearshift occurs, the available power is reduced to zero for all or part of the current and subsequent sectors until the specified gearshift time has elapsed. This provides some hysteresis to avoid the problem with the first method described above, where the drag-related drop in speed due to the interruption in power output would cause a lower gear to be selected, and a vicious circle ensues.

Combining Acceleration and Corners

We now an equation we can apply to curved sectors to tell us the maximum speed a car could negotiate the corner based on the limiting grip of the tyres, and an equation we can apply to a straight sector to tell us the exit speed from the sector. We don't have an equation that could tell us the exit speed of a sector when we the car is accelerating and travelling though a corner. It is necessary to know when the driver would have to lift off the throttle to control how much tyre grip is used for acceleration, and how much for cornering.

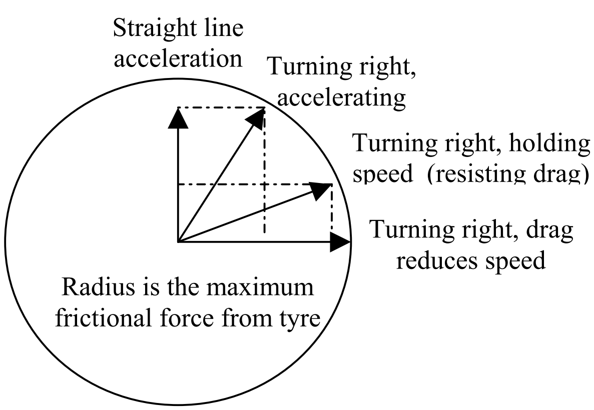
We also don't have an answer for how braking will take place, but we will deal with this later.

Consider the corner example introduced earlier and assume that the car arrives at the corner at just less than the maximum corner speed. The driver will not have to brake, but at some point in the corner will have to balance the car with the throttle. When the maximum frictional force from the tyres is reached, the accelerative force from the engine will need to be reduced to allow the tyres to generate the centripetal force required to turn the corner. If the driver does not lift off the throttle, then the car will not go around the corner. This is generally a bad thing.



We can use the diagram below to visualise the forces being transmitted by the tyre. The radius of the circle is the maximum frictional force μR.

While accelerating in a straight line, the force produced by the engine (longitudinal acceleration) will often be less than the maximum frictional force. Exceptions to this might be a grid start, or a Formula One car running on cold tyres. It should therefore be possible to negotiate a corner whilst still accelerating.



As the speed increases, the centripetal force (lateral acceleration) increases, and there will come a point where the result of the two perpendicular forces equals the maximum frictional force. We are now at the limit of the tyre's grip.

As the speed increases still further, the accelerative force will have to be reduced to ensure that the total force remains equal to the maximum frictional force. We are balancing the car on the throttle.

Given a constant radius corner, the speed can be increased in this way until the point is reached where the drag force is equal to the accelerative force from the engine.

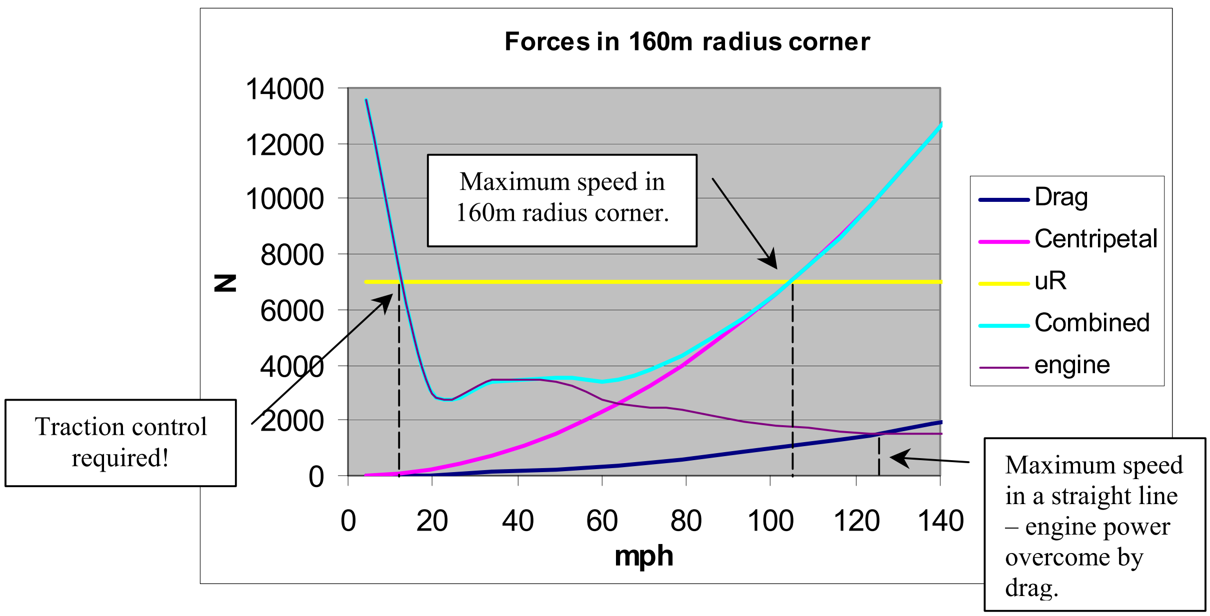
As an aside, the only way to get a larger centripetal force to turn in a smaller radius would be to gently reduce the engine power – which would result in the car being slowed down by the drag. We could not apply the brakes since this would introduce a longitudinal force, and coming suddenly off the throttle would have a similar effect due to 'engine braking'.

Putting it all together

Currently, we have equations for the 'straight line acceleration' and 'turning right, holding speed' conditions on the previous diagram. We need to have a method to calculate how much of the force available to accelerate the car can be transmitted through the tyres, in order that we can calculate the speed at the end of any track sector, curved or not.

It seems that the only way to do this will be to determine if the accelerative force from the engine needs to be limited. This condition will only occur when the total forces on the tyre are equal to the maximum frictional force – whether during low speed acceleration, or during high speed cornering.

The diagram above shows a graph of the combined longitudinal and lateral (engine and centripetal) forces acting on a car at different speeds in a corner with a radius of 160m.



At slow speed, the available force from the engine would exceed the maximum frictional force and cause wheelspin if all the power was applied. At higher speed, it can be seen that the centripetal force becomes the dominating factor. The maximum speed is reached when the combined force is equal to the maximum frictional force. We can also see from the diagram that the maximum straight-line speed would be reached when the drag force becomes equal to the accelerative force from the engine.

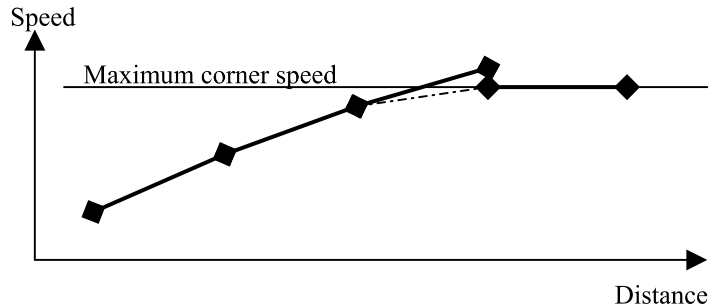
For the range of speeds in between, the ability of the car to accelerate is unaffected by the cornering forces, and hence we can use the straight-line acceleration equation to calculate the exit speed from the sector. The only proviso might be that we should calculate the distance travelled as the arc between the two points rather than the straight-line distance.

Slow speed wheelspin

Currently, we model traction control by limiting the longitudinal acceleration force from the engine to the maximum frictional force. This could potentially cause incorrect results for modelling acceleration from slow corners with increasing radius, but this is not thought to be significant. If it does prove to be significant, then the amount of centripetal force required at the same time will need to be taken into account.

Transition to cornering

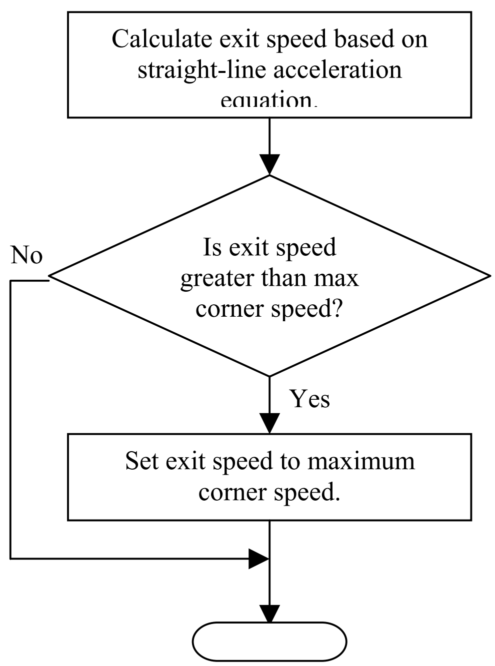
Modelling the transition from accelerating to cornering on the limit is fairly straightforward. The diagram below shows a car accelerating through a constant radius corner until it reaches the maximum corner speed.



In reality, the driver would continue to accelerate until the maximum corner speed was reached, then hold that speed. As our track is split up into sectors, we can model this as shown in the diagram, where the exit speed is simply set to the maximum corner speed. The average acceleration will be the same regardless of how we model the transition, since the entry and exit speeds will be fixed.

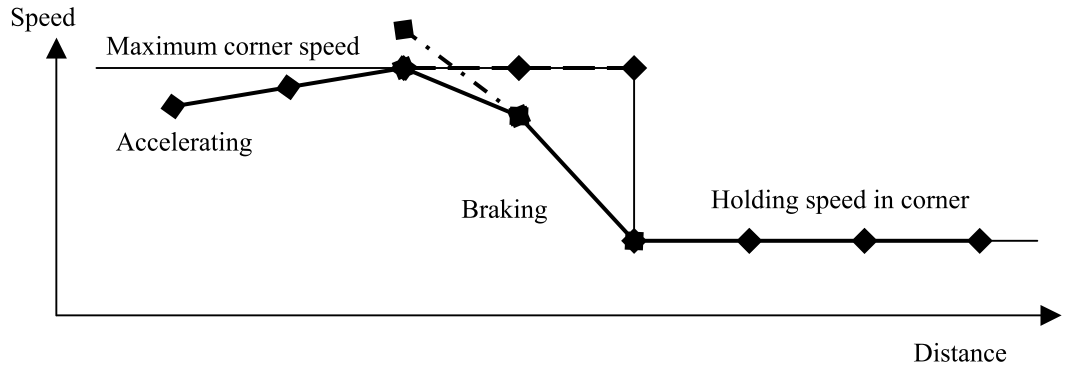
We can use the flow diagram below to select the correct exit speed.

The acceleration through the sector, the elapsed time and the amount of power consumed can all be calculated if the entry and exit speeds for the sector are known.



Braking

After accelerating down a straight on the racetrack, sooner or later, the car is going to come to a corner. If the corner is tight enough, the driver will need to slow the car down in order to get around the corner. The speed of the car will need to be reduced to the maximum corner speed by the time that the car arrives at the corner. This requires an element of forward planning that we have not considered so far.



The initial calculation of maximum corner speed and forward acceleration can be done by progressing forwards through each sector in turn, feeding the exit velocity from one sector into the entry velocity of the next. To calculate the best possible braking performance, we will first calculate the acceleration from rest and maximum corner speeds, then apply the braking forces on a second pass, this time going backwards through the sectors.

For the first pass, it will be clear that we will have to slow down for corners in some way or other. The simplest way to do this seems to be as follows. When considering a new sector, the entry speed is compared against the maximum corner speed for the sector. If the entry speed is higher, then the car would have needed to brake to this point. We reset the corner entry to a reasonable value – the maximum corner speed.

On the second pass, we can go backwards through the sectors and compare the entry speed for sector N+1 with exit speed for sector X. If they are different, then we will need to apply the brakes retrospectively during sector X. We take the entry speed for sector N+1 as the desired exit speed of sector N, and calculate the maximum speed from which it would be possible to decelerate under the cornering conditions for sector N. Since we are going backwards through the data, this may be thought of as acceleration. If this newly calculated entry speed for sector N is less than the existing corner entry speed, then we will use it, and will need to continue the braking during sector N-1. If the newly calculated entry speed is greater than the existing corner entry speed, then we will keep the existing corner entry speed and keep the new exit speed – this will have been the first sector where braking is required.

Either way, it will be necessary to determine how much braking force can be applied. This force will be what is left over from the maximum frictional force generated by the tyre after the centripetal force required to get around the corner is taken into account.

Force available for braking

The force available for braking (Fb) from the tyres at any point can be calculated as follows, if the total frictional force Ft and the centripetal force Fc are known.

$$Ft^2 = Fc^2 + Fb^2$$

$$Fb^2 = Ft^2 - Fc^2$$

$$Fb = \sqrt{Ft^2 - Fc^2}$$

We know that the maximum frictional force $Ft = \mu R$

$$Fb = \sqrt{\mu^2 R^2 - Fc^2}$$

We have a more complicated equation to consider for the centripetal force Fc, since this component is itself dependent on velocity. We need then either to attempt to consider the change of velocity during the sector, or opt for a less accurate method where a fixed speed is chosen from either the entry or exit speed. The simplest method will be to use the exit speed from the sector as this is known. This will mean that the centripetal force will be slightly under-estimated – as the exit speed is lower than the entry speed – but this is not thought likely to be a significant error.

The centripetal force for the exit speed will be $Fc = mv^2/r$

$$Fb = \sqrt{\mu^2 R^2 - (mv^2/r)^2}$$

$$Fb = \sqrt{\mu^2 R^2 - m^2 v^4/r^2}$$

Another factor contributing to the deceleration of the car will be the drag. This acts on the body and not the tyres, so it is in addition to any deceleration from the tyres. Again, we will use the drag at the exit speed of the sector – which will be a slight underestimation. You will recall that the drag force Fd can be calculated with this equation.

$$Fd = Cd \times \tfrac{1}{2}\rho v^2 \times A$$

The total decelerative force Fs will therefore be

$$Fs = Fd + Fb$$

$$Fs = \tfrac{1}{2}\rho v^2 ACd + \sqrt{\mu^2 R^2 - m^2 v^4/r^2}$$

The deceleration from this force will be $-a = -Fs/m$

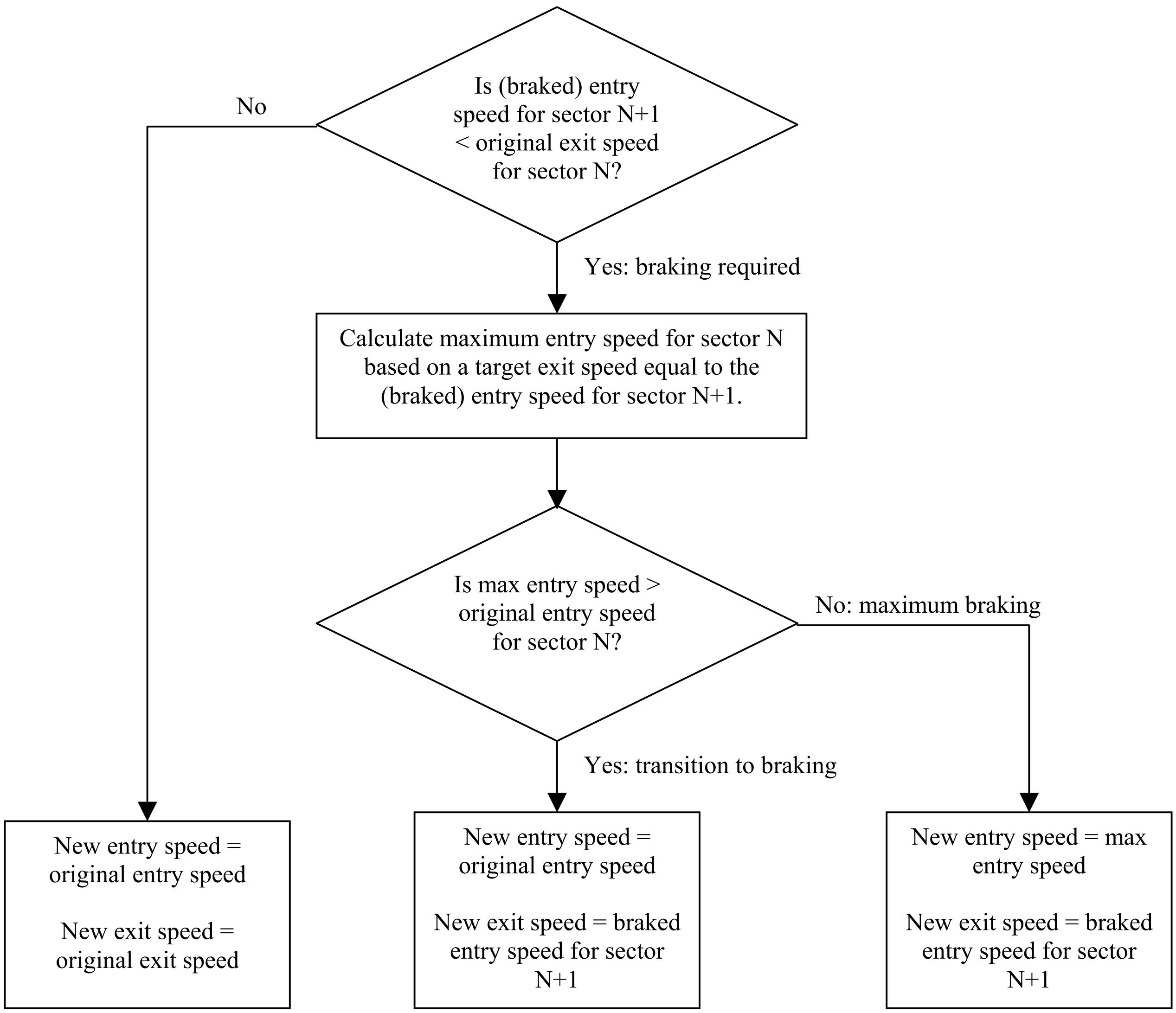
Thus the maximum entry speed u from which we could have braked would be:

$$u^2 = v^2 - 2as$$

$$u^2 = v^2 - 2(-Fs/m)s$$

$$u = \sqrt{v^2 + 2sFs/m}$$

Flow diagram for braking



Derived information

When a model has been built which can model acceleration from rest, gear changes, braking and cornering, it should be possible to obtain a useful piece of data – the lap time. It will also be possible to generate graphs of the car behaviour that can be compared with real data from telemetry systems.

A typical data logging system would allow the following parameters to be displayed, against distance or time:

- Longitudinal acceleration – relative to G
- Lateral acceleration – relative to G
- Engine RPM
- Wheel speed
- Gear
- Throttle position
- Steering angle

Displaying telemetry data

We can attempt to replicate the output of a data logging system, in order that results from the computer model and real results can be compared.

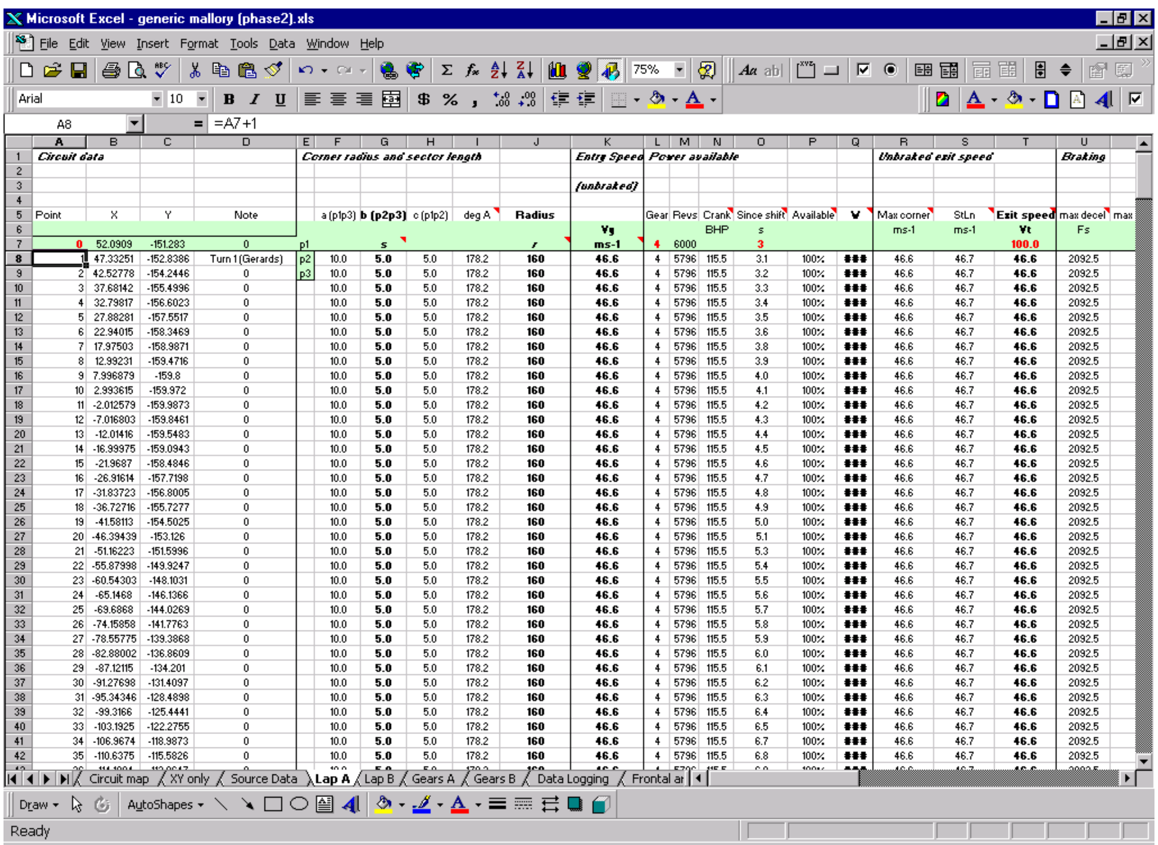
Parameter	How is it calculated?	Equation
Elapsed time	From average speed through the current sector and the sector length, plus the sum of all previous sector elapsed times.	
Distance travelled	From the sum of all previous sector lengths.	
Longitudinal acceleration	From the difference in entry and exit speeds, and the sector length.	
Lateral acceleration	By using the average sector speed in the centripetal acceleration equation.	
Engine RPM and gear	Both are already calculated in order to get the engine power output.	
Wheel speed	Exit speed for each sector is used, scaled to give an MPH figure rather than ms-1.	
Throttle position	Power required to accelerate the car and overcome drag, as a percentage of the available engine power. Shown as zero during braking.	
Steering Angle	Taken from 180-A (where A is the angle between two sectors).	

Output from Model

The graph below shows output from the model, based on the following inputs:

Item	Value	Note
Track	Generic Mallory circuit	
Mass of car + driver	437 + 75 = 512 kg	Based on Royale RP31M FF1600
Gear change time	0.3s	Target from 'Data Power'
Frontal Area	1.54m^2	From 'Race and Rally Car Sourcebook'
Cd	0.525	From 'Race and Rally Car Sourcebook'
Tyre μ	1.4	From 'Tune to Win' and 'Competition Car Downforce'
Power output	Max 124 BHP	Graph data from Dyno run of 1600cc Ford Cross-flow Mono-Kent engine. Gears from Royale's Hewland Mk.9
Transmission efficiency	91%	From 'Competition Car Downforce'
Density of air, ρ	1.23 kg/m3	From 'Data and Formulae'

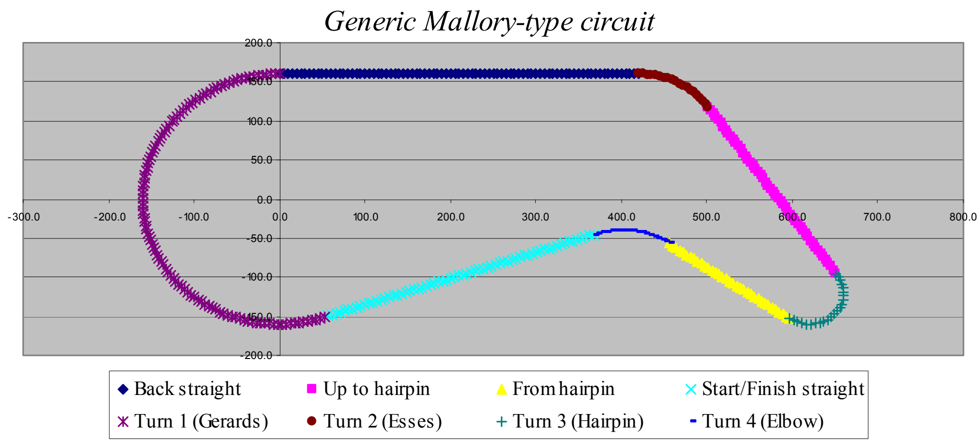
The model has been built up using Microsoft Excel – it might perhaps be useful eventually to create a model written in C, but the facilities for graph plotting and simple equation editing in Excel make this a good choice for the time being.



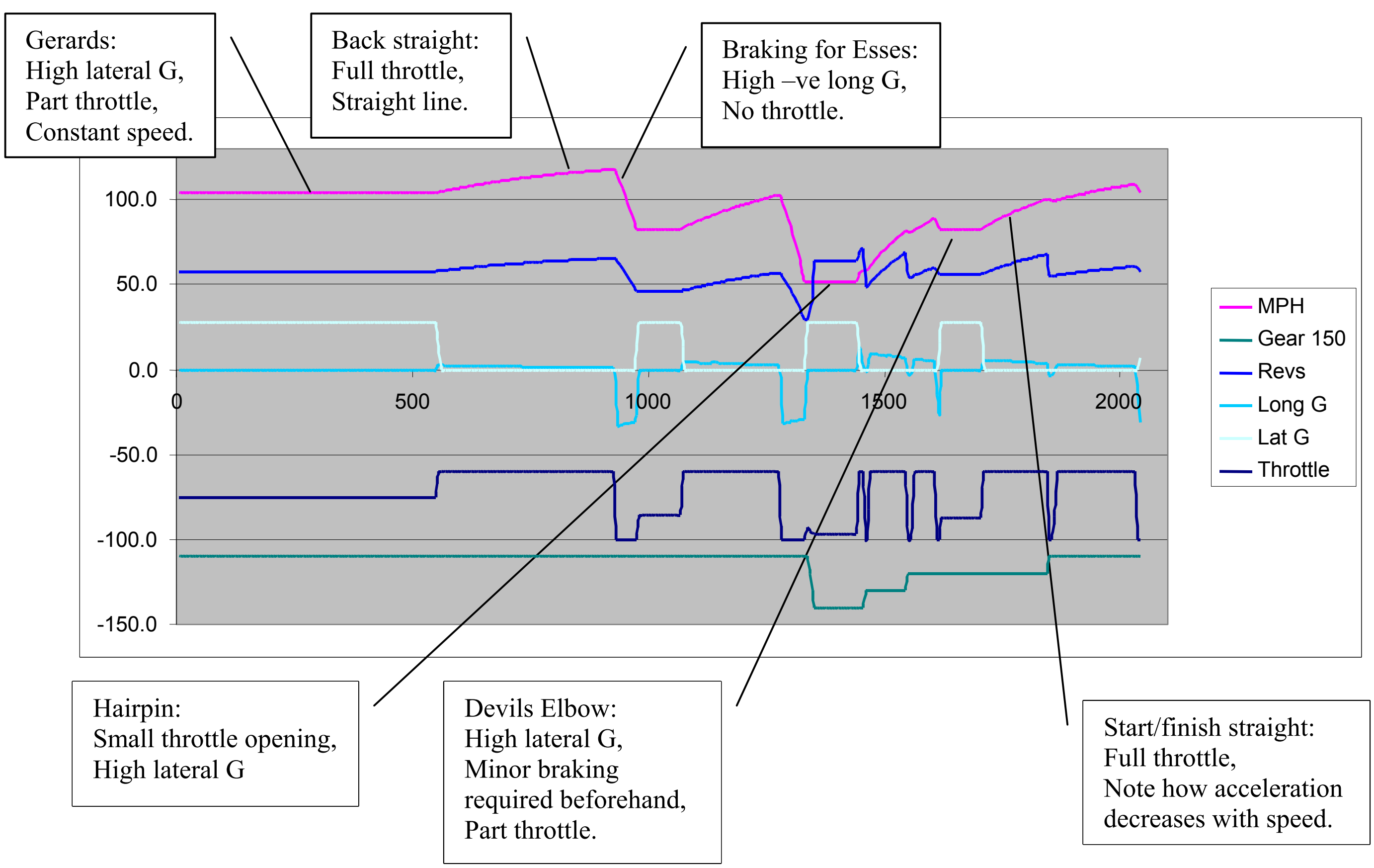
It is something of a pain to input the co-ordinates for the track data – and to potentially switch between data for different circuits. It's also not possible to run the model with different inputs automatically and create plots of lap time against mass, for example. This would be a good reason to re-create the model in C, to allow circuit data from a data logging system to be input directly into the model.

Generic Mallory Park type circuit

The circuit being used in this case is a very simple model, with straight straights and constant radius corners:



A simple plot of the data from the model against distance travelled is shown below:



We can see that the driver is making the best use of the tyres in this example under both cornering and braking. As this simple track has constant radius corners, the lateral G loading and throttle opening are constant throughout the corners – since the maximum corner speed remains constant. There are no corners where braking is required whilst cornering in this case – all braking is done in a straight line. The hairpin appears to be the only corner where downshifting is required.

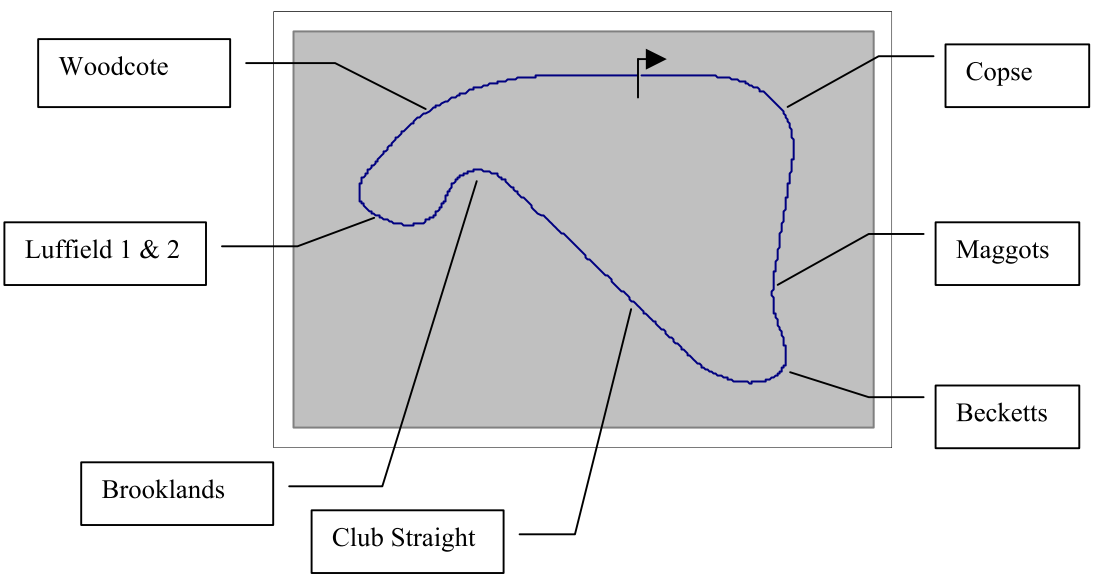
For the above example, the following results were also obtained:

Item	Value
Lap time	49.5s
Max Speed	118 mph
Min Speed	52.3 mph

Silverstone National Circuit

Now that we have a model that appears to create useful results, we can attempt to use it with some real track data. It has been possible to obtain the XY co-ordinates for the Silverstone National circuit, circa 1993 from a demonstration version of the PI Club Expert data logging system. The map data is stored in a form that made it very easy to reverse engineer. This demonstration system also provides real data for a car travelling around the circuit that we can use to compare against the model.

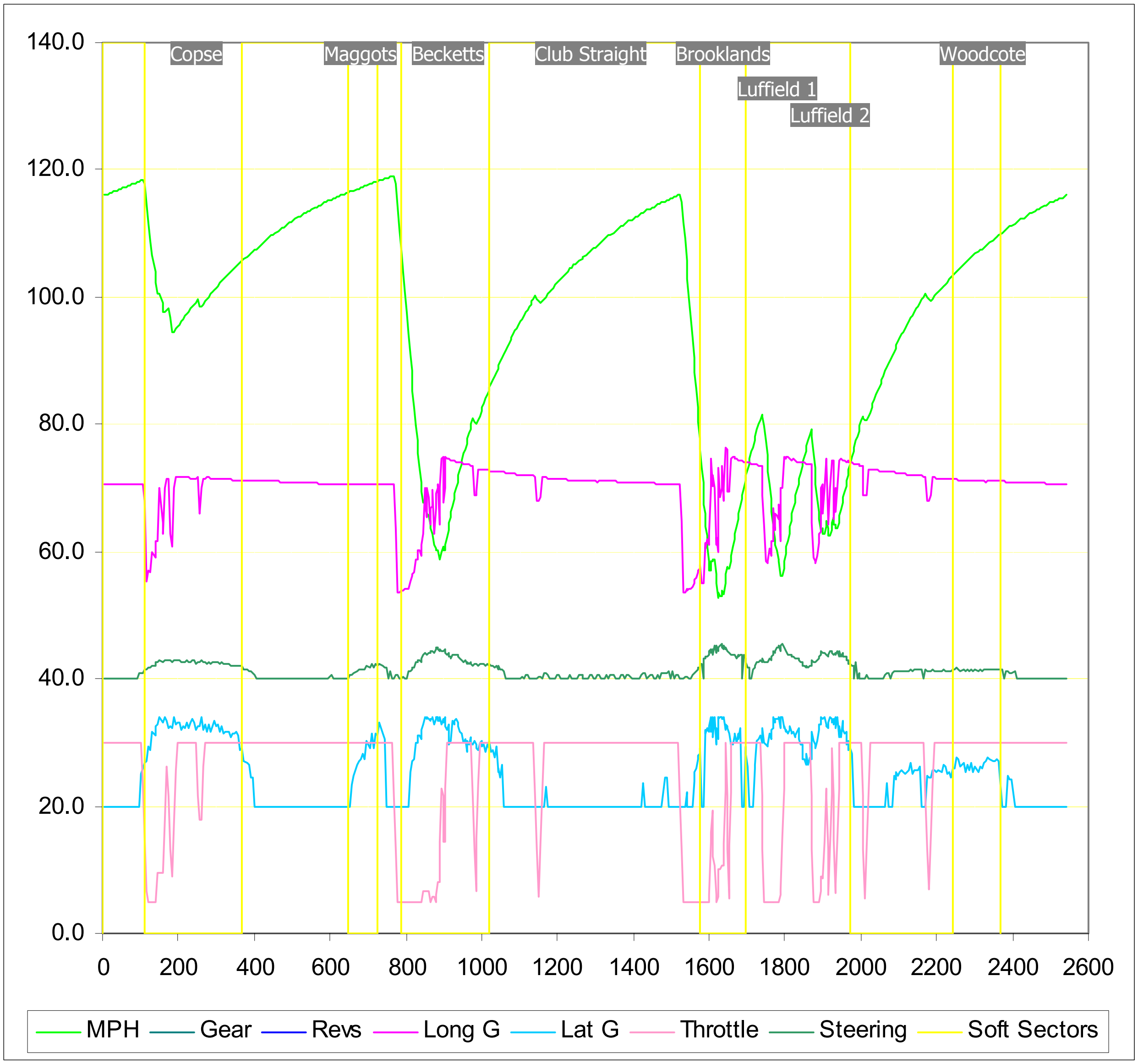
The map is created from the speed and lateral G recorded by the car as it travels around the circuit. It is the actual path travelled by the car around the circuit – the racing line.



The circuit data from Silverstone contains corners whose radius reduces up to the apex, then increases as the driver tries to take the best line on the available track. It also contains corners where braking is required as the car is turning a corner (between Brooklands and Luffield 1).

It has proved necessary to smooth the track data to some extent – although the coordinates are good enough to produce a smooth looking map, the calculations in the model produce the corner radius numbers which have a lot of noise. For this reason, the calculated radius at any point is averaged with the points before and after – a very simple way to clean up the data.

The plot below shows data from the model. It is in a slightly different form to the previous graph – this has been done to match the format of the data printed from the PI analysis software.

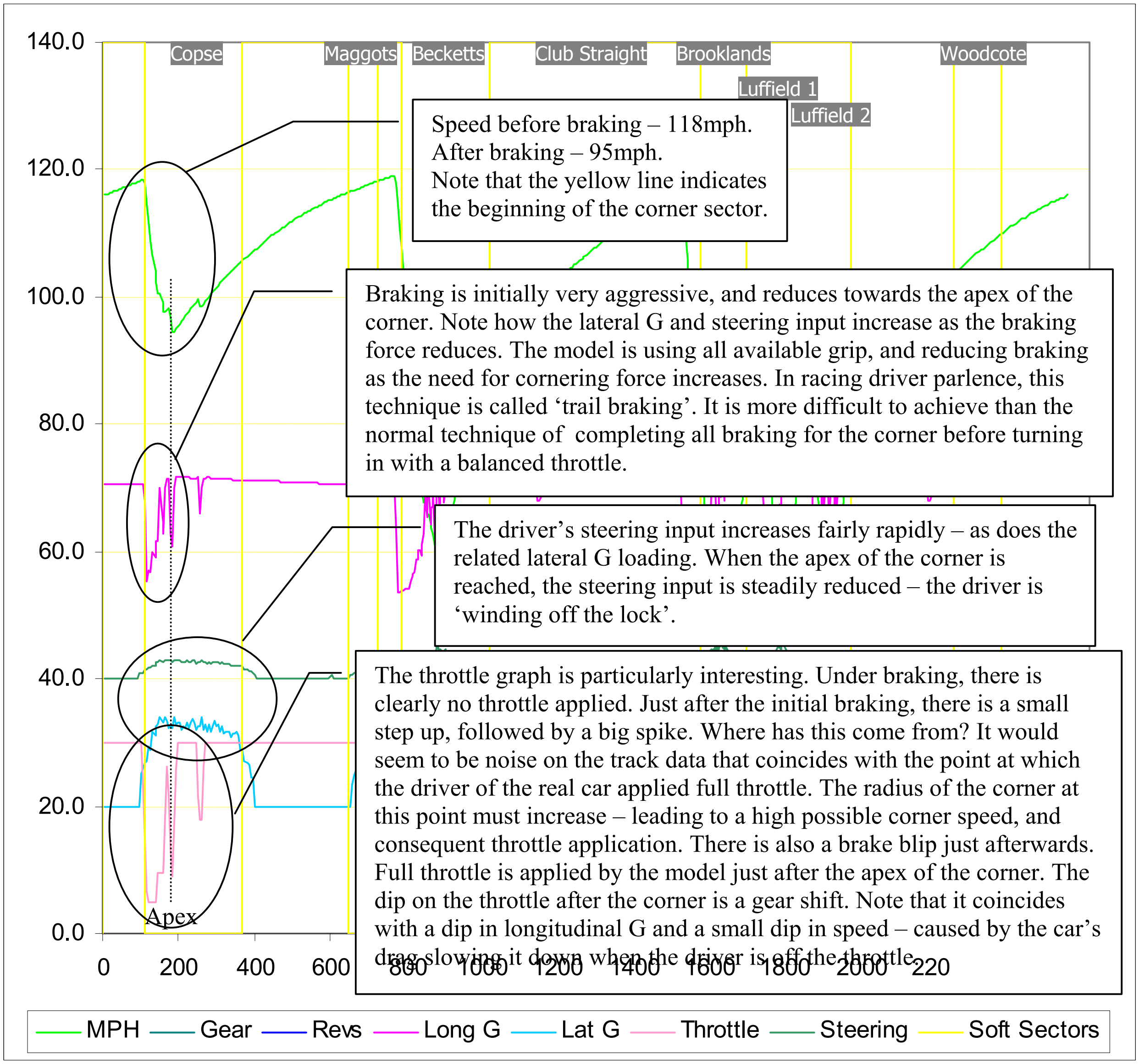


It should be noted that the lateral G and steering graphs only go in one direction, since the model cannot currently distinguish between left-hand and right-hand corners! This would only be a problem if we wanted to directly compare the results with real car data.

Also note that although the 'Gear' and 'Revs' lines are shown on the key, the actual traces has been omitted for reasons of clarity. The vertical yellow lines are used to show different sectors of the track – the start and end of corners can be set up on the software. This is useful since we can see where particular activity takes place relative to the start or end of corners.

There is a huge amount of useful information to be had from this kind of data logging trace – but it does require a good deal of looking to extract meaning from the graphs. In order to see if the model is working correctly, we can go through the traces corner by corner to see what is happening.

Silverstone – Copse

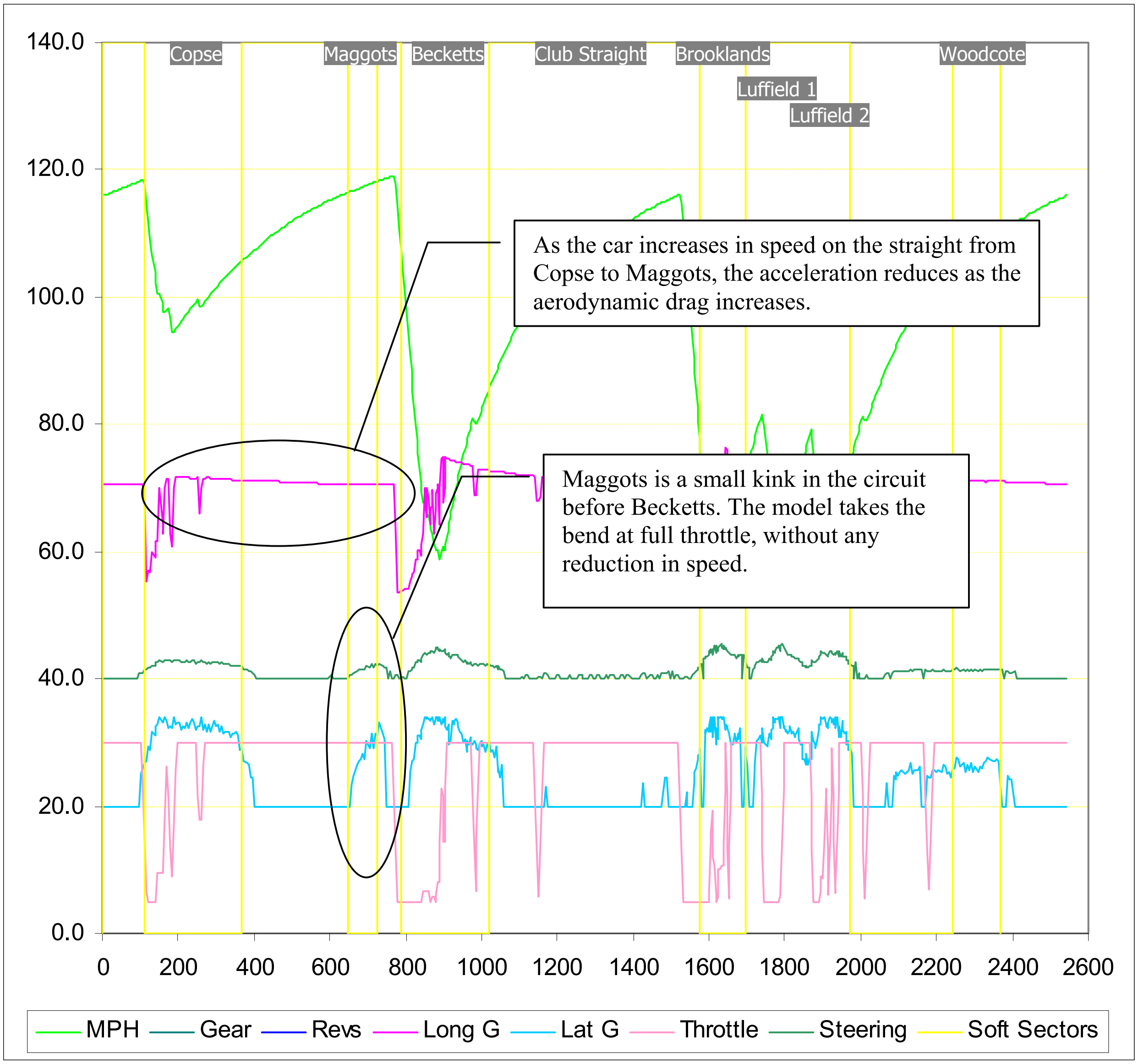


It would seem from the traces above that the model represents a super-human being! It has the ability to completely accurately judge how much grip is available, and trail brake to the apex of corners before applying the throttle. It cannot currently look very far forwards, and balance out the small variations in corner radius in the recorded data – a real driver would be unlikely to stab at the accelerator and then immediately touch the brake in the middle of a corner! Smoothness is usually the order of the day.

In the real datalogging traces, the driver gets most of the braking done before the corner entry point, and turns in on a balanced throttle, reducing the brake pressure as the steering input is increased. Full power is applied at the apex. The steering trace seems much more straight-ahead than the model – it looks as if the car has taken up a slip angle as it goes through the bend somewhat sideways. A note with the real trace indicates the traces were taken on a damp day.

Another trace from a demonstration version of Stack's data logging software for the Silverstone GP circuit does have a steering input pattern similar to the one from the model. In addition, there is much less understeer and oversteer in the Stack data, perhaps indicating that the weather was better when the data was recorded!

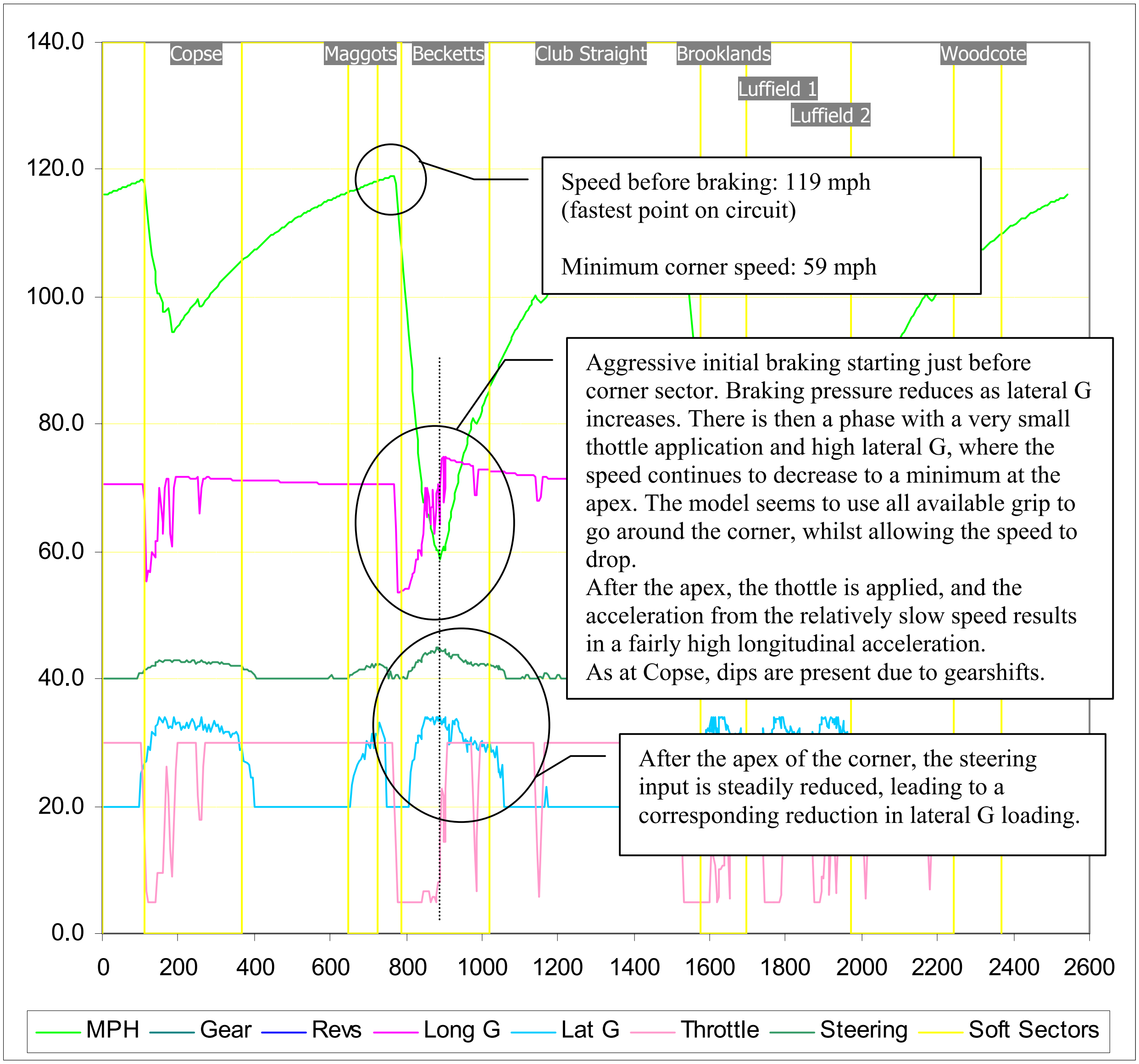
Silverstone – Maggots



In reality, Maggots presents a challenge for the driver since it is on the brow of a slight hill, and there is a large amount of braking to be done before Becketts – which is hidden over the hill. It is very temping to lift of the throttle on the entry to Magotts.

Silverstone – Becketts

Becketts is a corner with a very tight entry, and a lot of space on the exit.



In the real data trace, the driver does most of the braking prior to the corner, and turns in with a balanced throttle. The steering lock and throttle remain fairly constant throughout the short corner, after which full throttle is applied and steering lock reduced. Incidentally, the real data shows serious power-on oversteer on the exit of this slow corner!

Silverstone conclusions

Having considered the output of the model, we can draw the following conclusions:

- The model represents a superhuman driver who can extract the maximum grip from the tyres at any point. A good target to aim for, but perhaps not a driving technique the club racer playing the percentages game would want to emulate!

- The model appears to be affected to some greater or lesser degree by artefacts on the data which describes the path of the car, although these are not immediately visible on the map data. Perhaps a good map would be one from a very smooth lap – perhaps not the lap with the quickest time, but one where the optimum line is taken in the smoothest way.

- The model appears to work! Of course, we have to remember that many important factors are not modelled – such as suspension and tyre behaviour, weight transfer and the acceleration and deceleration of the rotating masses such as wheel/brake disc/driveshaft assemblies.

Scenarios

Mass, engine power, shift time.

Wings, downforce and drag

Now we have created a model that appears to be producing useful results, we can start to consider what benefit might be obtained by adding some downforce producing devices to the car. Typically this will be in the form of wings, but other aerodynamic devices such as splitters, spoilers, underfloor tunnels and diffusers could be considered.

We can consider again the forces in a 160m radius corner introduced earlier, with and without a wing of area $0.4m^2$ being added to the car.

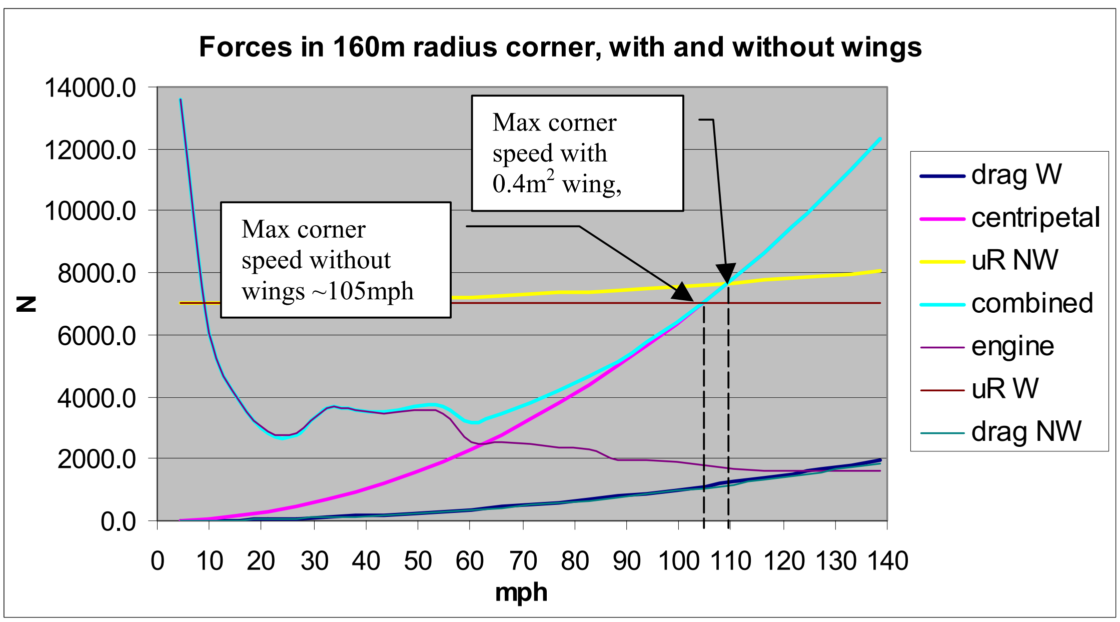
The downforce generated by a wing is dependent upon the plan area of the wing and the coefficient of lift (negative lift in our case):

$$F_L = C_L \times \tfrac{1}{2}\rho v^2 \times A$$

This can be added to the normal force as a result of the mass of the car (mg) in the limiting friction equation:

$$\begin{aligned} F &= \mu R \\ &= \mu mg + \tfrac{1}{2}\mu\rho v^2 A \end{aligned}$$

As we would expect, the limiting friction will increase with the square of speed. It can be seen from the diagram below that the maximum corner speed will thus increase, in the case shown by about 5mph for the $0.4m^2$ wing.



The wing will add extra drag to the car, again this is calculated using the coefficient of drag and the plan area of the wing:

$$F_d = C_d \times \tfrac{1}{2}\rho v^2 \times A$$

Although this is relatively small compared with the drag due to the frontal area, we will nevertheless consider the drag associated with the wing in our calculations. As a side point, in *Competition Car Downforce*, it is suggested that the drag from the front wing can be ignored.

Maximum corner speed with wings

We can now attempt to calculate the maximum corner speed (the crossover point in the previous diagram). This equation for the wingless case was previously used:

$$(\mu R)^2 = v^4((m^2/r^2) + ({}^1/_4\rho^2 A^2 C_D{}^2))$$
$$= v^4((m/r)^2 + ({}^1/_2\rho A C_D)^2)$$

If we refer to frontal area as A_F (previously A) and wing area as A_W, the wing coefficient of drag as C_{WD} and the car's coefficient of drag as C_{FD} (previously C_D) then we can add in the wing drag component:

$$F_D = {}^1/_2\rho A_F C_{FD}v^2 + {}^1/_2\rho A_W C_{WD}v^2$$
$$= {}^1/_2\rho(A_F C_{FD} + A_W C_{WD})v^2$$

$$(\mu R)^2 = v^4((m/r)^2 + ({}^1/_2\rho(A_F C_{FD} + A_W C_{WD}))^2)$$

The normal force at velocity v, where the wing coefficient of lift is C_L will be:

$$R = mg + {}^1/_2\rho A_W C_L v^2$$

So now we can attempt to solve for v, by first getting rid of the square on the left hand side:

$$\mu R = v^2 \sqrt{((m/r)^2 + ({}^1/_2\rho(A_F C_{FD} + A_W C_{WD}))^2)}$$

$$\mu mg + {}^1/_2\mu\rho A_W C_L v^2 = v^2 \sqrt{((m/r)^2 + ({}^1/_2\rho(A_F C_{FD} + A_W C_{WD}))^2)}$$

$$\mu mg = v^2 \sqrt{((m/r)^2 + ({}^1/_2\rho(A_F C_{FD} + A_W C_{WD}))^2)} - {}^1/_2\mu\rho A_W C_L v^2$$

dividing both sides by v^2

$$\mu mg/v^2 = \sqrt{((m/r)^2 + ({}^1/_2\rho(A_F C_{FD} + A_W C_{WD}))^2)} - {}^1/_2\mu\rho A_W C_L$$

$$v^2 = \frac{\mu mg}{\sqrt{((m/r)^2 + ({}^1/_2\rho(A_F C_{FD} + A_W C_{WD}))^2)} - {}^1/_2\mu\rho A_W C_L}$$

$$v = \sqrt{\frac{\mu mg}{\sqrt{((m/r)^2 + ({}^1/_2\rho(A_F C_{FD} + A_W C_{WD}))^2)} - {}^1/_2\mu\rho A_W C_L}}$$

This certainly is an equation that could do with some tidying up, but we could use it in its current form in the model. It is however hard to relate it back to the equation used before wings are applied, or even when the wing area is zero!

Straight line speed, with wings

The equation without wings was as follows, when re-written to use our new terminology:

$$v = \sqrt{u^2 + 2s((P/u) - ½\rho u^2 A_F C_{FD})/m}$$

Adding the additional drag component:

$$v = \sqrt{u^2 + 2s((P/u) - ½\rho u^2 (A_F C_{FD} + A_W C_{WD})/m}$$

This will make the car slower in straight line, as the wing area or coefficient of drag increases.

Maximum braking force, with wings

The wings should also increase the braking capacity of the car since the normal force on the tyres and thus the maximum limiting frictional force is being increased.

For the model, we calculate the maximum braking force to be applied throughout the sector. Clearly as speed decreases, the normal force associated with the wings will also decrease, however note that we calculate the force available only at the end of the sector, so the braking effort will be slightly underestimated.

The equation for maximum braking force without wings consists of a portion related to the force available after the cornering has be accounted for, and a pure drag portion. It was as follows, when re-written to use our new terminology:

$$F_S = ½\rho v^2 A_F C_{FD} + \sqrt{\mu^2 R^2 - m^2 v^4/r^2}$$

Introducing the additional drag of the wing:

$$F_S = ½\rho v^2 (A_F C_{FD} + A_W C_{WD}) + \sqrt{\mu^2 R^2 - m^2 v^4/r^2}$$

Substituting our new value for the normal force R, which now includes the downforce produced by the wing, we get our revised equation:

$$F_S = ½\rho v^2 (A_F C_{FD} + A_W C_{WD}) + \sqrt{\mu^2 (mg + ½\rho v^2 A_W C_L)^2 - m^2 v^4/r^2}$$

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